MODELLING AND SIMULATION OF MULTIPLE-RESISTANCE CONDUCTIVE HEAT TRANSFER PHENOMENA

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ABSTRACT

An important aspect of chemical engineering curriculum is to teach one/multi-dimensional conduction, convection and radiation processes of heat transfer. To give better illustration of these concepts, a computer simulation tool is very important. This computer simulation tool acts as an additional teaching aid for visualizing the complex transport processes, and therefore is helpful in facilitating and enhancing learning development. The objective of this report is to develop mathematical programming of multiple-resistance conductive heat transfer phenomena using a mathematical software package, i.e., MATLAB. Mathematical software package can expose the students to theoretical and practical features of heat transfer. Furthermore, the definition of conduction is the transfer of energy through matter from particle to particle. It is a heat transfer directly through materials, occurring when mass of material stays in place. It is the transfer and delivery of heat energy from atom to atom within a substance. Conduction is most effective in solids but it can also happen in fluids. An experiment was conducted to measure the temperature along the surface of the pin fin assuming steady state heat transfer, free convection and the tip of the fin insulated. The experiment was repeated with different materials having different values of thermal heat conductivity (k) and a graph was plotted between temperature variance with length and time of the rod. The results obtain thought the simulation numerical coding and the experiments were verified by using the MATLAB Tool. Those data of results are obtaining through estimation and imagine via no experimental procedure evidence proof. Thus, a comparative study was done for each of the materials to see the temperature variation along the length and time of the rod in different cases.

Key words: *Conductive Heat Transfer, MATLAB, Modelling, Simulation*

ABSTRAK

Satu aspek penting dalam bidang kejuruteraan kimia adalah untuk mengajar pengaliran sementara satu/multi-dimensi, perolakan dan sinaran proses pemindahan haba. Untuk menyampaikan konsep-konsep ini, alat simulasi komputer amat diperlukan kerana alat tersebut merupakan sebagai alat bantuan dalam aktiviti pembelanjaran dan pengajaran (P & P) “bonus” untuk menggambarkan proses pengangkutan yang kompleks. Alat itu juga dapat membantu dalam memudahkan dan meningkatkan pembangunan pembelajaran. Objektif bagi penyelidik ini adalah untuk membangunkan matematik dan pengaturcaraan bagi pelbagai rintangan konduktif fenomena pemindahan haba, iaitu MATLAB, yang boleh mendedahkan pelajar kepada teori dan praktikal pemindahan haba. Pengaliran adalah pemindahan tenaga melalui kira dari zarah untuk zarah. Ia adalah pemindahan haba secara langsung melalui bahan-bahan , yang berlaku apabila jisim bahan berada di tempat . Konduktif adalah pemindahan dan pengagihan tenaga haba dari atom ke atom dalam bahan. Pengaliran adalah yang paling berkesan dalam pepejal tetapi ia boleh berlaku dalam cecair. Satu eksperimen telah dijalankan untuk mengukur suhu sepanjang permukaan sirip pin menganggap keadaan mantap pemindahan haba, olakan bebas dan hujung sirip terlindung. Eksperimen diulangi dengan bahan-bahan yang berbeza mempunyai nilai yang berbeza kekonduksian haba (*k*) dan graf akan dilukis antara perbezaan suhu dengan panjang dan masa. Keputusan mendapatkan fikir kod berangka simulasi dan eksperimen telah disahkan dengan menggunakan MATLAB. Oleh itu satu kajian perbandingan telah dilakukan untuk setiap satu daripada bahan-bahan untuk melihat perubahan suhu di sepanjang panjang dan masa rod dalam kes-kes yang berbeza.

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LIST OF ABBREVIATIONS

$\dot{E}_g$	Energy Generation
$\dot{E}_{in}$	Energy Inflow
$\dot{E}_{out}$	Energy Outflow
$\dot{E}_{st}$	Energy Storage
$\overline{Nu_g}$	Rayleigh number
$\frac{\mathrm{d}E_{st}}{\mathrm{dt}}$	Rate of energy stored within the control volume
T_a^f	Final Atmospheric Temperature,K
$\frac{dQ}{d\theta}$	Rate of Flow of Heat
$\frac{dt}{dx}$	Rate of change of temperature in direction x
A	Area, m^2
C_p	Specific Heat Capacity, J $kg^{-1}K^{-1}$
D/d	Diameter, m
h_c	Convective Coefficient
h_f	Latent Heat of Fusion, kJ/kg
k	Thermal Conductivity, Wm^{-1} K^{-1}
L	Length, m
P	Perimeter, m
Q	Rate of Heat Transfer, kJ min^{-1}
S	Shape Factor
T	Temperature, K
t	Time, h
$T_{u/\infty}$	Air Temperature, K

U	heat transfer conductance, $Wm^{-2}K^{-1}$
W	Power, J/s
x, y, z	Cartesian Coordinates

Greek symbols

ρ	density, kg m^{-3}
α/a	Thermal diffusivity

Subscripts and superscripts

c	Cold fluid
cold	Cold end
h	Hot fluid
hot	Hot end
Ambient	Ambient

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Programming, modelling and simulation are useful to demonstrate the solutions of multifaceted engineering problems. Among the available marketable packages to achieve the objective are *Aspen Plus*, *Aspen HYSYS*, *Scilab*, *MATLAB*, *Microsoft Excel* and so on. Sinclair J.L (1998) stated that integrating computational software such as *Computational Fluid Dynamics* package into teaching of undergraduate transport courses is an effective way to expose learning and teaching progress to conceptual and practical examples of fluid-particle flow. This is the way; the student can better visualize the flow behaviour, and discover the effect of changes in system geometry, properties and operating conditions. Other researchers like D.J. Brown *et al.* (1997) also applied Computational Fluid Dynamics to simulate the temperature profiles inside furnace tube wall. The importance of computational techniques in many capacities of chemical engineering is undoubted. It acts as an educational tool in design, modelling and simulation of chemical processes. H.O. Kassim and R.G. Cadbury (1996) discussed the role of computer in chemical engineering education. One of the ways to increase the students' interests in programming is by integrating computational modelling with underlying key elements of the theory, on which the model is based. They believed that programming approach could indoctrinate a logical and disciplined approach to design problems.

Additionally, it is a common to acknowledge that real-life processes often deviate from idealist. Awareness and ability to apply programming skills to solve transient heat transfer processes would be of great advantage for the undergraduate students to challenge the real-life complication when they work as engineers in future. With this knowledge, it is hope that these future graduates will not only be the users but also creators of technology.

After discussing on the importance and benefit of using computational techniques and mathematical software packages, now discuss about the definition of heat transfer.

Nowadays, heat transfer is still an active area of study and application in many of engineering sectors. In recent years, an excessive deal of concentration has been established in the topics like contact resistance, which mean a temperature difference develops between two objects that have zero contact with each other. Additional issues of present interest comprehend non-Fourier conduction, where the processes take place in high-speed that the equation described below does not apply. On the other hand, the problems related to transport in miniaturized systems are saving a great of interest. Increased interest has also been directed to ways of handling composite materials, where the ability to conduct heat is very directional (Robert, 1999).

Heat transfer is an operation that occurs repeatedly in the many types of industry, for example in food industry, chemical industry, petrochemical industry and so on. Besides, heat transfer is a dynamic process in which heat is transferred spontaneously from one body to another cooler body. However, the rate of heat transfer depends to the differences in temperature between one or more than two bodies, the greater the difference in temperature, the greater the rate of heat transfer. Temperature difference between the source of heat and the receiver of heat is consequently the driving force in heat transfer. An increase in the temperature difference increases the driving force and therefore increases the rate of heat transfer. The heat passing through from one body to another end of the body travels through some medium or atom which in general offers resistance to the heat flow. Thus, temperature difference and resistance to heat flow are the factors which can affect the rate of heat transfer. This is the general equation which linked to these two factors, shown in Equation 1-1:

$$\text{Rate of transfer} = \frac{\text{Driving force}}{\text{Resistance}} \qquad \textbf{Equation 1-1}$$

For rate of heat transfer:

$$\text{Rate of Heat Transfer} = \frac{\text{temperature difference}}{\text{heat flow resistance of medium}} \qquad \textbf{Equation 1-2}$$

During processing, temperatures may modification and therefore the rate of heat transfer will also increase or decrease. This is called unsteady state of heat transfer and vice versa to the steady state heat transfer when the temperatures do not change. Unsteady state heat transfer is more complex since an additional variable, time, enters into the rate equations, for example of unsteady state heat transfer is the heating and cooling of cans in a retort to sterilize the contents..

In theoretical, heat can be transferred in three ways: by conduction, by radiation and by convection. In conduction, the molecular energy is directly exchanged, from the hotter to the cooler regions, the molecules with greater energy communicating some of this energy to neighbouring molecules with less energy. An example of conduction is the heat transfer through the solid walls of a refrigerated store. Next, as for convection, it is the transfer of heat by the movement of groups of molecules in a fluid. The groups of molecules may be moved by either density changes or by forced motion of the fluid. One of the examples of convection heating is cooking in a jacketed pan: without a stirrer, density changes cause heat transfer by natural convection; with a stirrer, the convection is forced. Meanwhile, last type of heat transfer is radiation. Radiation is the transfer of heat energy by electromagnetic waves, which transfer heat from one body to another, in the same way as electromagnetic light waves transfer light energy. The example of radiation is when a foodstuff is passed below a bank of electric resistance heaters that are red-hot. In general, heat is transferred in solids by conduction, in fluids by conduction and convection. Heat transfer by radiation occurs through open space, can often be neglected, and is most significant when temperature differences are substantial. In practice, these three types of heat transfer may occur together. For calculations it is often best to consider the mechanisms separately, and then to combine them where necessary (Earle, 1983).

Furthermore, conduction is the transfer of energy through matter from atom to atom. It is a heat transfer directly through materials, occurring when mass of material stays in place. It is also the transfer and distribution of heat energy from atom to atom within a substance. Many examples of heat transfer conduction in our daily lives. As in our daily lives, a spoon in a cup of hot soup becomes warmer because the heat from the soup is conducted along the spoon. Conduction is the most effective in solids and it can happen in fluids as well. In addition, conduction is definite heat transfer which define as of molecular agitation within a material without any motion of the material as a whole, as known as heat. If one end of a metal rod is at a higher temperature, then energy will be transferred down the rod in the direction of the colder end because the higher speed particles will collide with the slower ones with a net transfer of energy to the slower ones. For heat transfer between two plane surfaces, such as heat loss through the wall of a house, the rate of conduction heat transfer is:

$$\frac{Q}{t} = \frac{kA(T_{hot} - T_{cold})}{d} \qquad \textbf{Equation 1-3}$$

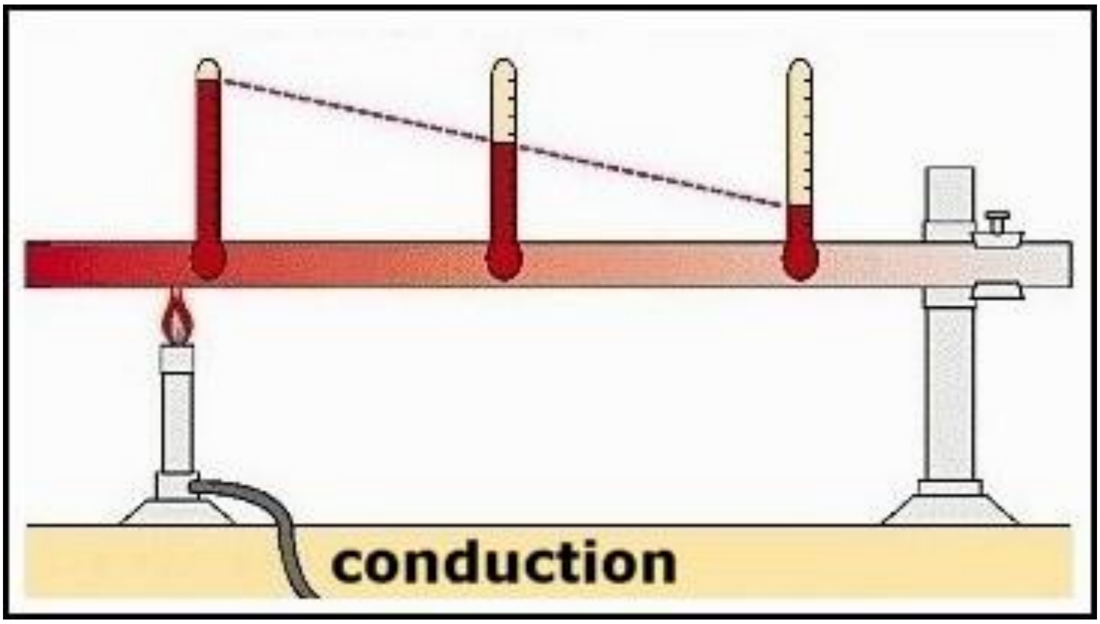


Figure 1-1 Mechanism of Conduction Heat Transfer Phenomena

For example, let think whether have ever perceived that metals tend to feel cold. Actually these metals are not colder. But actually these metals just only feel colder because they conduct heat away from our hand. Actually the heat is leaving hand as cold. This makes it clear that: what identifies heat is temperature difference and not temperature.

Conduction heat transfer phenomenon is encountered in many real life problems. One may consider the following few examples among the broad range of conduction problems. A heat sink containing of small devices is recommended to maintain the electronic components below a specified temperature. There are few devices of the size and material and configuration are necessitated:-

i) Liquid in Glass Thermometer: Liquids like mercury and alcohol are used to measure temperature by the skill of putting the liquid in glass. Determine the minimum and maximum temperature that mercury and alcohol can measure by this skill of thermometer engineering.
ii) Heat Loss of an Aluminium Alloy Plate: An aluminium alloy plate is suddenly quenched into liquid oxygen. Find the heat loss by the plate. Calculate the requisite time for the plate to reach to specific temperature.
iii) Nuclear Reactor Coolant Pump: The temperature of some nuclear elements begins to increase, if the coolant pump fails. Determine the required time that engineers need to fix the pump before disintegration happens.
iv) Space Vehicle Re-entry Protection: A heat guard is used to protect a space vehicle during re-entry. The protection ablates as it passes through the atmosphere. Specify the required protection thickness and material to protect a space vehicle during re-entry.
v) Supersonic Rocket Nozzle Protection: The throat of a supersonic rocket nozzle is protected by inserting a porous ring at the throat. Injection of helium through the ring lowers the temperature and protects the nozzle. Determine the amount of helium needed to protect a rocket nozzle during a specified trajectory.

Hence, this report intends to illustrate the linkage between theoretical concepts of one dimensional heat conduction and computer simulation by using MATLAB. The simulation tool acts as an additional teaching and learning aid for better illustration of the complex heat transfer process. This approach was introduced as one of the topics in the Undergraduate Research Project course whereby the student were given the opportunity to integrate their theoretical knowledge in heat transfer, numerical method and computing skills expanded a priori.

1.2 Motivation and statement of problem

Every day, the sun's thermal energy journeys through 91 million miles as known as 146 million km in SI units of empty space all the way to earth. The sun's energy is the driving force for all life systems that exist on our planet today. Without a way to transfer this energy from the sun to the earth, there will be in big trouble. And, this is just one example of why the transfer of thermal energy is such an important concept to humans. The earth wouldn't be what it is today if without a technique to transfer thermal energy. The sun's energy is radiated in the form of heat and visible light and also predominant source of renewable energies.

The physics behind the transfer of thermal energy explains how to attach renewable energy from the sun and convert it to energy that can use in daily lives. As mentioned earlier, the transfer of heat between substances at different temperatures occurs in three different ways: conduction, convection, and radiation. All these are part of everyday experiences. Conduction is the transfer of heat through a specific material, such as heat moving through a metal pot being heated on the stove. Convection is the transfer of heat through a fluid such as water or air, instead of a material. For example, when lying down on the floor and will feel the air down low is cooler; this is because warmer air rises towards the ceiling, leaving the cool air behind, low to the ground. Situations in which heat is transmitted through air are examples of convection. Radiation is energy that is radiated or transmitted in the form of rays, waves or particles.

The first law of thermodynamics delivers a useful tool for many heat transfer theories or problems. In expectation of such problems, general formulation of the first law is obtained. Consider applying energy conservation to the control volume shown in Figure 1-2.

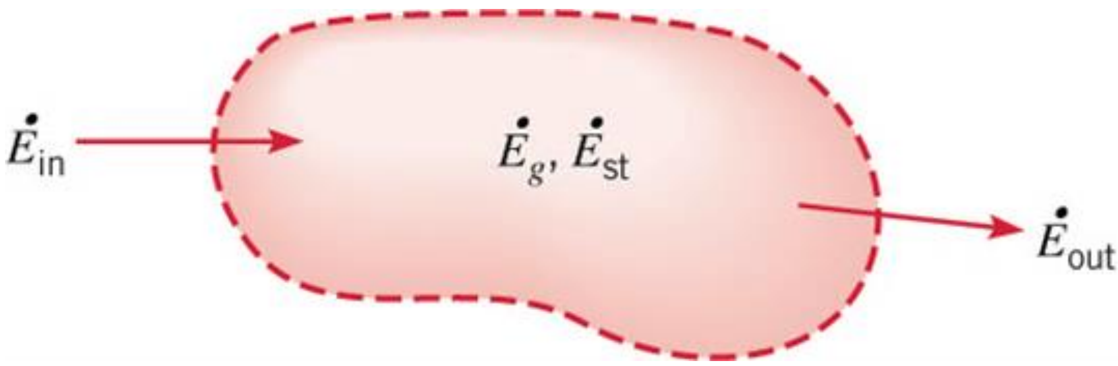


Figure 1-2 Conservation of Energy for a Control Volume

The first step is to identify the control surface by representation a dashed line. The next step is to identify the energy terms. At any instant, these include the rate at which thermal and mechanical energy enter and leave through the control surface, $\dot{E}_{in}$ and $\dot{E}_{out}$. Correspondingly, thermal energy may be generated within the control volume due conversion from other energy forms. Refer to this process as energy generation, and the rate at which it occurs is designated as $\dot{E}_g$.The rate of energy stored within the control volume, $\frac{\mathrm{d}E_{st}}{\mathrm{dt}}$, is designated as $\dot{E}_{st}$. A general form of the energy conservation requirement on the rate basis is given by

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$ **Equation 1-4**

where $\dot{E}_{in}$ and $\dot{E}_{out}$ (the inflow and outflow terms) are surface phenomena. That is, they are associated exclusively with processes occurring at the control surface and are proportional to the surface area. A common situation involves energy inflow and outflow due to heat transfer by conduction, convection, and radiation. For $\dot{E}_g$ is the energy generation term associated with conversion from chemical energy, electrical energy, electromagnetic energy, or nuclear energy to thermal energy. It is a volumetric phenomenon, which it occurs within the control volume and is proportional to the magnitude of this volume. For example, an exothermic chemical reaction may be occurring, converting chemical to thermal energy. The net effect is an increase in the thermal energy of matter within the control volume. Another source of thermal energy is the conversion from electrical energy that occurs due to resistance heating when an electric current is passed through the conductor. And, $\dot{E}_{st}$ is also a volumetric phenomenon, and changes within the control volume may be due to changes in the internal energy. The change in the internal energy consists of a thermal component which interpretations for the translational, rotational, or vibrational motion of the atoms or molecules comprising the matter; a chemical component, which accounts for energy, stored in the chemical bonds between atoms; a nuclear component, which accounts for binding forces in the nucleus; and a latent component, which relates to phase change between solid, liquid and vapour states.

In this research, the statement of the problem is profile a reliable but simple programming in order to study for conductive heat transfer phenomena for learning and teaching activity.

1.3 Objective

The main objective of this research is to develop mathematical and programming of multiple-resistance conductive heat transfer phenomena.

1.4 Scope of this research

The following are the scope of this research:

i) Mathematical modelling of multiple-resistance conductive heat transfer. This work is carried out by deriving the Fick's Law based on heat balance.
ii) Based on mathematical model, a programming is developed to understand the heat transfer phenomena.
iii) To measure the temperatures distribution along the length of rod allows determining the coefficient of thermal conductivity of material of which the rod is made.

1.5 Main contribution of this work

The main contribution of this report is into two sections, which is: -

i) Teaching and
ii) Lessoning.

1.6 Organisation of this thesis

The structure of the reminder of the thesis is outlined as follow:

Chapter 2 provides a description of the applications and general design features which is theoretical; develop of design and mathematical modelling. There are two fundamental rules leads to Fourier's Law of general heat conduction which is first law and second law of thermodynamics. First law of thermodynamics states that under steady conditions the rate of heat flow will be constant. Second law of thermodynamics also one of the fundamental rules which shows the direction of this flow is from the higher temperature to lower temperature surface. For develop of design, the effects Rayleigh number, *Prandtl* number: radius ration and boundary condition type on heat transfer were investigated. Two general techniques are applied in mathematical modelling, for example finite difference and finite element concepts.

Chapter 3 gives a review of the approach applied for heat conduction modelling and simulation of multi-phase which the equation that governs the heat flow is known as Fourier's Law, and in the axial direction it is written as $q_x = -kA_x \frac{dT}{dx}$. There were two types of conduction of heat, known as conduction of heat in axial direction and conduction of heat between bar and air. This chapter also will discuss the process flow of the MATLAB GUI programming run and the step to coding for simulation.

Chapter 4 is devoted to the result of experimental and simulation programming result for heat conduction.

Chapter 5 draws together a summary and recommendation of the thesis and outlines the future work which might be derived from the model developed in this work.

CHAPTER 2

LITERATURE REVIEW

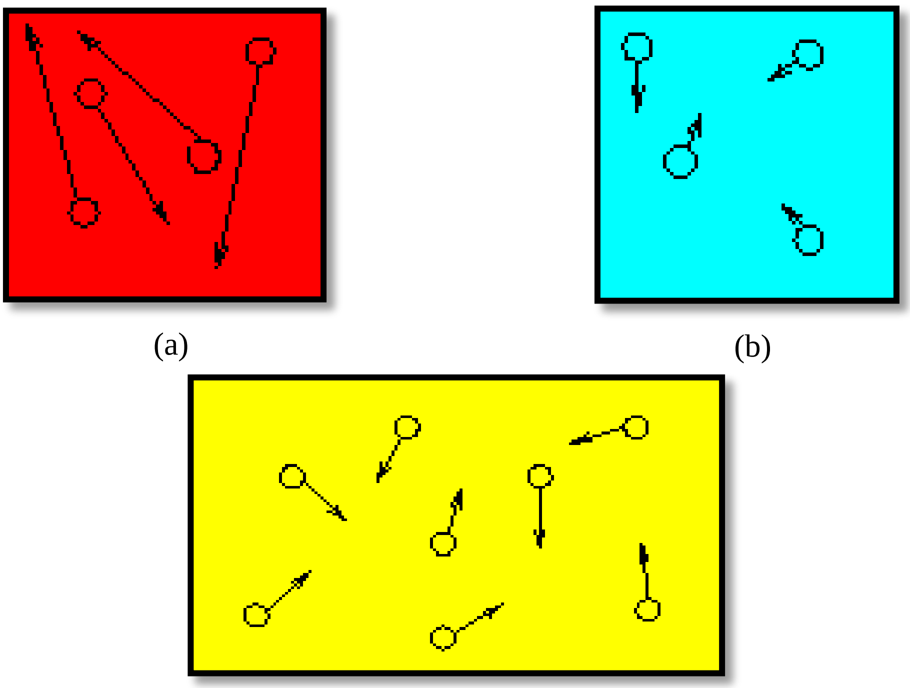
2.1 Overview

This report presents three parts of heat transfer conduction which is theoretical; develop of design, mathematical modelling and computer programming. There are two fundamental rules leads to Fourier's Law of general heat conduction which is first law of thermodynamics states that under steady conditions the rate of heat flow will be constant. Second law of thermodynamics also one of the fundamental rules which show the direction of this flow is from the higher temperature to lower temperature surface. For develop of design, the effects Rayleigh number, *Prandtl* number: radius ration and boundary condition type on heat transfer were investigated. Two general techniques are applied in mathematical modelling, for example finite difference and finite element concepts. In computer programming part, there was 2 programs was develop for the two-dimensional (2D) transient temperature profiles inside the tube wall, which are the first is the main program MTPROF.m and the second MATLAB function program TPROF.m.

2.2 Theoretical of Knowledge

Many of our daily experiences involve heat-transfer phenomena. Boiling, condensation, laundry hanging under the sun during a breezy day dries fast because the moisture evaporates and diffuses easily into the moderately dry moving air. All of these natural phenomena can be explained by using the theories of heat transfer. Since there are so many natural phenomena that happen in our daily lives can be explained by the theories of heat transfer, the chemical engineering student to learn the mass transfer is to apply the theories of heat transfer in our field and use it in our daily lives to help our people to achieve for quality lifestyle. Besides, this can also apply the understanding of heat transfer mechanisms such as conduction, convection and radiation for understanding the performance of the heat transfer that use in almost all chemical and related industrial.

The flow of heat by conduction occurs via collisions between atoms and molecules in the element and the subsequent transfer of kinetic energy. Let consider two substances at different temperatures separated by a barrier which is subsequently removed, shown as in the Figure 2-1 (Theoretical Physics at the University of Winnipeg, 1999).



(a) (b)

(c)

Figure 2-1 Heat transfer by conduction (a) Fast (Hot) Atom (b) Slow (Cold) Atom (c) Common Temperature

When the barrier is removed, the fast (“hot”) atoms collide with the slow (“cold”) ones. In such collisions, the faster atoms lose some speed and the slower ones gain speed; thus, the fast ones transfer some of their kinetic energy to the slow ones. This transfer of kinetic energy from the hot to the cold side is called a flow of heat through conduction.

As the first law of thermodynamics states that under steady conditions, the rate of heat flow will be constant. Second law of thermodynamics shows that the direction of this flow is from the higher temperature surface to the lower one. These two fundamental rules lead to Fourier’s law of general heat conduction:

$$\frac{dQ}{d\theta} = -kA\frac{dt}{dx} \qquad \textbf{Equation 2-1}$$

where $\frac{dQ}{d\theta}$(quantity per unit time) is the rate of flow of heat, “A” is the area at right angles to the direction in which the heat flows, and $-\frac{dt}{dx}$ is the rate of change of temperature with the distance in the direction of the flow of heat, i.e., the temperature gradient. The factor “k” is called the thermal conductivity; it is a characteristic property of material through which the heat is flowing and varies with temperature.

Equation 2-1 is used as a basis for derivation of the unsteady-state three-dimensional energy equation for solids and static fluids:

$$c\rho\frac{\delta t}{\delta\theta} = \frac{\delta}{\delta x}\left(k\frac{\delta t}{\delta x}\right) + \frac{\delta}{\delta y}\left(k\frac{\delta t}{\delta y}\right) + \frac{\delta}{\delta z}\left(k\frac{\delta t}{\delta z}\right) + q' \qquad \textbf{Equation 2-2}$$

where x, y, z are distances in the rectangular coordinate system and q’ is the rate of heat generation by chemical reaction, nuclear reaction, or electric current in the solid per unit of volume. Solution of Equation 2-2 with appropriate boundary and initial conditions will give the temperature as a function of time and location in the material. Equation 2-2 may be transformed into spherical or cylindrical coordinates to conform more closely to the physical shape of the system.

Different materials transfer heat by conduction at different rates; this is measured by the material's thermal conductivity. For example, place a material in